REDIMAT
Journal of Research in Mathematics Education

Hipatia Press
www.hipatiapress.com

Instructions for authors, subscriptions and further details:

http://redimat.hipatiapress.com

# Students' Recognition of Function Transformations' Themes Associated with the Algebraic Representation

Wajeeh M. Daher[1], & Ahlam A. Anabousy[2]

1) Al-Qasemi Academic College of Education and An-Najah National University, Israel.
2) Al-Qasemi Academic College of Education & Tel-Aviv University, Israel.

Date of publication: June 24th, 2015
Edition period: June 2015-October 2015

---

**To cite this article:** Daher, W.M., & Anabousi, A.A. (2015). Students' recognition of function transformations' themes associated with the algebraic representation. *REDIMAT*, *Vol 4*(2), 179-194. doi: 10.4471/redimat.2015.1110

**To link this article:** http://dx.doi.org/10.4471/redimat.2015.1110

---

*REDIMAT, Vol. 4 No. 2 June 2015 pp. 179-194*

# Students' Recognition of Function Transformations' Themes Associated with the Algebraic Representation

Wajeeh M. Daher
*An-Najah National University*

Anlam A. Anabousi
*Tel-Aviv University*

(*Received: 23 May 2014; Accepted: 11 June 2015; Published: 24 June 2015)*

**Abstract**

The topic of function transformations is a difficult mathematical topic for school and college students. This article examines how students conceive function transformations after working with GeoGebra, when this conceiving relates to the algebraic representation. The research participants were 19 ninth grade high achieving students who learned, with the help of GeoGebra translations, reflection and stretch. During their learning, the participants worked with transformations on the absolute function, the cubic function, and the quartic function. After they finished the transformation unit, the participants solved mathematics problems by means of function transformations. The research findings show that the participants were generally able to solve successfully mathematical problems, by means of transformations on new and non-basic functions. Furthermore, the participants encountered difficulties in working with translations. Future researches could examine the impact of activities that include such functions and that are GeoGebra based on students' conceptions and behavior when performing translations is involved.

**Keywords:** Function transformations, algebraic representation, translations, reflections, stretch

2015 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2015.1110

*REDIMAT, Vol. 4 No. 2 June 2015 pp. 179-194*

# Reconocimiento de los Estudiantes de las Transformaciones de Funciones Asociado a la Representación Algebraica

Wajeeh M. Daher
*An-Najah National University*

Anlam A. Anabousi
*Tel-Aviv University*

(*Recibido: 23 May 2014; Aceptado: 11 June 2015; Publicado: 24 June 2015)*

**Abstract**

Las transformaciones de las funciones es un tema difícil de matemáticas tanto para estudiantes en la escuela, como en la universidad. En este artículo examinamos cómo los estudiantes conciben las transformaciones de las funciones después de trabajar con GeoGebra, cuando esta concepción se relaciona con la representación algebraica. Los participantes en la investigación fueron 19 niños/as de noveno curso con un promedio alto, que estaban aprendiendo, con ayuda de GeoGebra, traslaciones, reflexiones y prolongaciones. Los participantes trabajaron con transformaciones sobre funciones absoluta, cúbica y cuadrática. Cuando terminaron la unidad sobre funciones, resolvieron problemas de matemáticas usando transformaciones de funciones. Los resultados muestran que generalmente fueron capaces de resolver de manera satisfactoria los problemas de matemáticas, usando transformaciones de funciones nuevas y no-básicas. Futuras investigaciones podrían examinar el impacto de las actividades que incluyen este tipo de funciones en GeoGebra que están basadas en las concepciones de los estudiantes y en su comportamiento cuando involucran transformaciones.

**Keywords:** Transformación de funciones, representación algebraica, traslación, reflexión, prolongación

2015 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2015.1110

School students generally meet the topic of function transformations at middle school, but they meet the transformations there as a tool for learning the quadratic function, not as a subject learnt for its own sake. This absence of the function transformations topic, as an independent subject, from the school curriculum could be due to the difficulty met by school students when learning it (Eisenberg & Dreyfus, 1994; Zazkis, Liljedahl & Gadowsky, 2003). This reality made us want to try teaching the whole subject of function transformations in the middle school, specifically to grade nine students, utilizing the potentialities of new technologies, specifically Geogebra. Our attempt seemed to us interesting because we intended to involve the participating students with transformations of functions that are not usually taught in the middle school, as the cubic and the quartic functions. The transformations of these functions are far less accessible to students than the basic ones (Eisenberg & Dreyfus, 1994), where the cubic function was mentioned specifically by the previous study as thus. Being aware of this situation, we expected that the potentialities of GeoGebra; a new technological tool suggested in the last year as a tool for the learning of mathematics, would help middle school students have access to the transformations of non-basic functions. We wanted also to examine how middle school students who performed transformations on cubic and quartic functions perform these transformations on new more complicated function like the rationale function. We are interested in this article in students' conceiving of function transformations after they used GeoGebra to learn the subject of function transformations. Specifically, we are interested in middle school students' conceiving of transformations on some 'complicated' functions; specifically the absolute value of a quadratic function and the rationale function.

## Literature Review

Researchers have been interested in students' conceptions of functions' transformations for approximately two decades now (Eisenberg & Dreyfus, 1994; Consciência & Oliveira, 2011). Generally, researchers examined students' conceptions of basic functions, usually the quadratic function. Doing so they tried to characterize students' learning and difficulties when learning function transformations, comparing usually between students' conceptions of the vertical translation and their conceptions of the horizontal transformation. Doing so, they pointed at students' difficulties in conceiving the horizontal translation. One such research is that of Eisenberg and Dreyfus (1994). They reported that after six lessons on function transformations using computer software, the students' were not successful in (1) dealing with higher order polynomials, (2) visualizing a horizontal translation in comparison to a vertical one, suggesting that reason for the difficulty could be the more complicated visually processing of the horizontal transformation. Almost one decade later, Zazkis, Liljedahl and Gadowsky (2003) tried to examine the explanations given by secondary school students and secondary school teachers to a translation of a function, focusing on the parabola $y = (x-3)^2$ and its relationship to $y = x^2$. The participants' explanations focused on patterns, locating the zero of the function, and the point-wise calculation of function values. The results showed that the horizontal shift of the parabola is, at least at the beginning, inconsistent with the participants' expectations and counterintuitive for most participants.

Another study that showed the problematic treatment of function transformations by students is that of Lage and Gaisman (2006). They interviewed university students while solving problems involving transformations of functions. The results showed that few students could work confidently with transformation problems, where their work demonstrated that they had not interiorized the effects of transformations on functions when it was needed to think in terms of co-variation of the dependent and independent variables of the function. Specifically, students had troubles when they had to identify which transformation had been applied to a particular basic function. When a transformation was given, they had problems finding its properties. All these difficulties were more apparent when the representation used in the question was graphical.

McClaran (2013) points at another common finding in studies of students' understanding of function transformations, namely their dependence on memorized rules for transformations in order to perform them. For example they memorized rules for vertical and horizontal translations, being more concerned with remembering rules than with understanding the behavior (Zazkis, Liljedahl & Gadowsky, 2003). This reliance of the students on memorized rules or procedures results in their lacking of conceptual understanding of function transformations (Kimani, 2008).

Few researchers attempted to study students' learning of function stretching. One such a study is that of Sever and Yerushalmy (2007) who described the first attempts of two calculus students to understand the concept of stretching of functions using technological tools, where each of the students was engaged in interpreting dynamic graphs in order to deal with graph stretching in various situations. The authors emphasized the influence of technology on students' learning, saying: "the tool aroused an on-line sensory stimulus through which they could act in a tangible and concrete way on the abstract functions" (p. 1518).

## Research Rationale and Goals

As described above, being aware of students' difficulties with the topic of function transformations, we wanted to examine their conceiving of this topic with new technologies, specifically Geogebra for its constructing capabilities. We expected these capabilities to support students, in our case middle school students, in their conceiving of function transformations. The results of the research would shed light on the potentialities and capabilities of new technologies, specifically GeoGebra, when students use it to learn function transformations, a topic described by various researchers as difficult for school and university students (Eisenberg & Dreyfus, 1994; Zazkis, Liljedahl & Gadowsky, 2003).

In this sense, our research question may be formulated as:

> What are the conceptions of middle school students of transformation on non-basic and new functions after working in an interactive mathematics environment?

## Research Methodology

### Research Context

We conducted the research in a middle school, and specifically with grade 9 excellent students. The function transformations' unit was taught by the second author to grade 9 students using Geogebra, which is a relatively new technological tool for teaching and learning several mathematical topics. The unit was composed of five lessons, where each lesson consisted of 90 minutes. The first lesson reviewed the use of transformation in real life contexts, as well as the main characteristics of the three non-basic functions: $y=|x|$, $y=x^4$ and $y=x^3$. The second and third lessons treated the horizontal and the vertical translations respectively. The fourth lesson treated the reflection transformation, while the fifth lesson treated the stretch and compression transformations. Carrying out the transformation activities, the students described the relations between the three representations of the transformations, specifically when the algebraic rule of a function was given or when the graph of a function was given. Furthermore, during performing the activities, the teacher worked as a facilitator of students' learning, directing them and requesting them to justify their answers. All the activities were following the exploration strategy, i.e. designed to encourage the students discover the properties of the transformations, as well as the relations between their themes, with the help of technology, in our case Geogebra.

The students were engaged in different activities, but the emphasis was on the algebraic and graphical representations of functions. The participants were 19 excellent grade 9 students who had different individual abilities in mathematics. We decided to work with excellent students taking into consideration previous studies' results regarding the difficulties that students confront when they learn transformations, even when the functions are basic ones like the quadratic functions.

## Data Collecting Tools

We collected the data from students' answers on two questions that evaluated students' conceiving of function transformations. We describe these tasks and their right answers below, in the item ‘The task.’

## Data Analysis Tools

To analyze the collected data we used deductive content analysis (Zhang & Wildemuth, 2009). This analysis is involved in performing constant comparisons between the units of gathered data (verbal sentences, graphs, algebraic rules or a combination of them) in order to categorize them in terms of themes related to the different transformations (e.g. the type of the translation, the size of the translation, the direction of the translation, etc.).

## Description of the Task

We gave the participants two questions, where these questions involved the relation of the algebraic representation of the function transformations with other representations of these transformations, namely the verbal and the graphical. The transformations in the two questions dealt with non-basic functions that were new to the participating students.

In the first question, we gave the students the function $f(x) = |x^2 - 5x + 4|$ and its graph shown in Figure 1. This question had three parts, where part 1 dealt with the translation transformation, while part 2 dealt with the reflection transformation, and part 3 dealt with the stretch transformation.

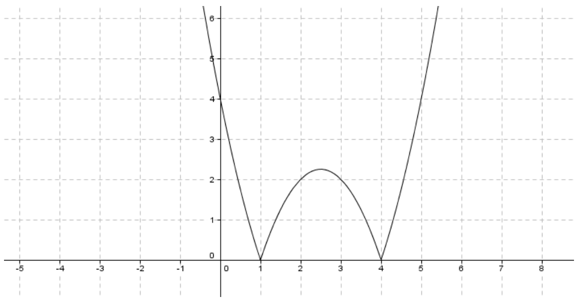


*Figure 1.* The given graph of the given function $f(x) = |x^2 - 5x + 4|$

Part 1 of the first question had two items: (a) we want to translate the given graph three units horizontally to the left and two units vertically below, and write the algebraic rule of the resulting function $f(x) = |x^2 - 5x + 4|$ to the following function $f(x) = |(x + 1)^2 - 5\ (x + 1) + 4| + 2$.

The correct answers of this part of the first question are given below:

a) $f(x) = |(x + 3)^2 - 5(x + 3) + 4|$-2
b) Translating the graph of the original function one unit horizontally and two units vertically.

Part 2 of the first question also had two items: (a) *We want to draw the graph of the function* $-f(x)$; as well as we asked the following question (b) *Would you like to write the algebraic expression of the resulting function?*

The correct answers of this part of the first question are given below:

a) The one showed in the attached graph (figure 2)

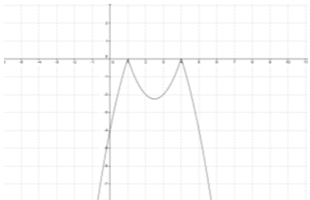


*Figure 2.* The given graph of the given function $-f(x)$

b) $f(x) = -|x^2 - 5x + 4|$

Part 3 of the first question had two items too: (a) *We want to draw the graph of the function* $k(x) = 1/3 f(x)$; and the question (b) *Would you like to write the algebraic expression of the resulting function?*

Again, the right answers of this part of the first question are given below:

a) The graphical representation showed in figure 3

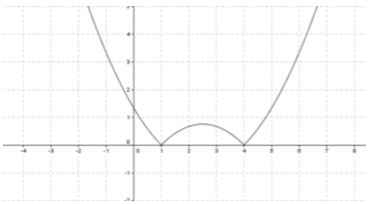


*Figure 3.* The given graph of the given function $k(x) = 1/3 f(x)$

b) The function $k(x) = 1/3\ |x^2 - 5x + 4|$

The second question differed from the first one in treating the three types of transformations together. We gave the participants the algebraic rule of the original function $f(x) = 1/(1 + x^2)$, as well as its graph as in figure 4. The students were required to write the algebraic rule of the transformed function.

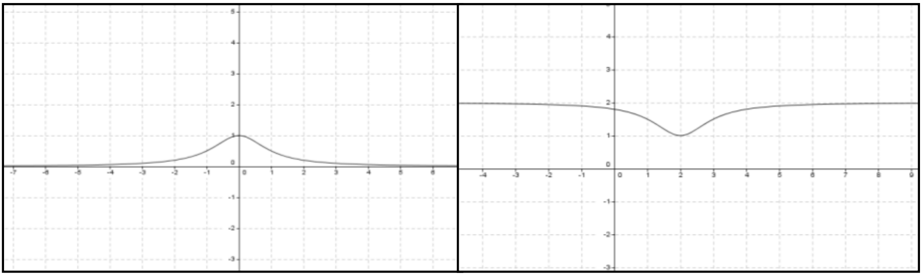


*Figure 4.* The original function $f(x) = 1/(1 + x^2)$ and the transformed one

The correct answer to the second question is:

$$f(x) = -1/(1 + (x - 2)^2) + 2$$

## Findings

### Recognizing the Algebraic Meanings of Verbal Expressions Associated with Translations and Vice Versa – The Case of the Function $f(x) = |x^2 - 5x + 4|$

Answering the first item of part 1 of the first question, the students needed to perform two transformations: the vertical and the horizontal translation. Performing the vertical transformation, fourteen students knew the algebraic meanings of the verbal expressions in terms of transformations. Two students did perform a vertical transformation but to the top. Three students performed arithmetic operations that did not fit the vertical transformation, such as multiplying 4 and 2, writing $|\ldots + 8|$in place of $|\ldots + 4| - 2$.

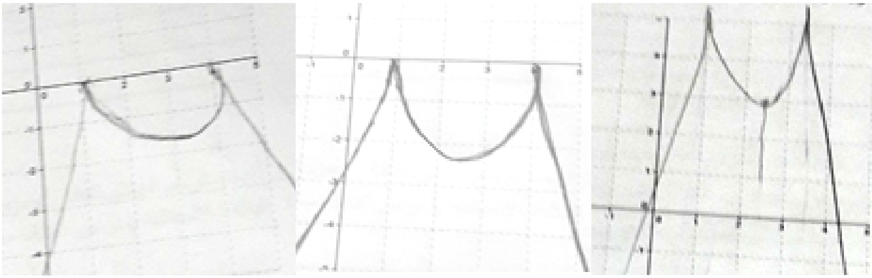
Performing the horizontal transformation, six students recognized the algebraic meanings of the verbal actions. They wrote different correct answers, such as $f(x) = |x^2 + x - 2| - 2$, $f(x) = |(x+3) - 5(x+3) + 4| - 2$, $f(x) = |(x+3)^2 - 5(x+3) + 4| - 2$. The rest did not know the algebraic meanings of the verbal horizontal translation. Seven students subtracted 3 from the expression inside the absolute value, instead of subtracting 3 from x, while three students added 3 to the expression inside the absolute value, instead of subtracting 3 from x. Two students did not recognize the horizontal transformation; i.e. neglected it.

Answering the second item of part 1 of the first question, eight students recognized the verbal meanings of the algebraic operations in terms of transformations. Further, eight students did not recognize some of the verbal meanings of the algebraic operations associated with the translation. One student of the eight did not recognize the verbal meaning of the horizontal translation. Three students of the eight did not recognize the size of the horizontal translation, while two students of the eight did not recognize the direction of the horizontal translation. The rest of the eight students did not recognize the size of the vertical transformation. Further, the last two students did not answer the question.

## Recognizing the Graphical and Algebraic Meanings of the Algebraic Operations in Terms of the Reflection Transformation: The Case of the Function $f(x) = |x^2 - 5x + 4|$

The first item of part 2 of the first question involved recognizing the graphical meanings of multiplying a function with (-1); i.e. what graphical change is needed on the graph of the original function as a result of this multiplication. The second item of part 2 of the first question involved recognizing the appropriate algebraic change in this case.

Answering the first item of part 2, five students recognized the graphical meanings of the algebraic action, drawing the graph of the transformed function correctly, as in figure 5 (a). Eleven students recognized the graphical meanings of the algebraic action, but they performed it inattentive to the intersection point of the graph of the function with the y-axis, though they were attentive to the intersection points of the graph with the x-axis. Figure 5 (b) shows one of their graphs. One student did not recognized the reflection axis involved in the transformation, making it y=3 (near the extreme point of the middle part of the graph) instead of y=0. This student's graph is shown in figure 5 (c). The last two students did not answer the question.



*(a)* *(b)* *(c)*

*Figure 5.* Students' recognition of the graphical meaning of the algebraic operations on the original function

When answering the second item of part 2, sixteen students knew the algebraic meanings of the algebraic action, while one student performed the multiplication inside the absolute value, writing $f(x) = |-x^2 + 5x - 4|$, being inattentive to the meaning of the absolute value. Another student multiplied (-1) with x instead of the value of $f\ (x)$, writing $f(x) = -x$. The last student did not answer the question.

### The Graphical and Algebraic Meanings of the Algebraic Actions in Terms of Stretch/Compress Transformation: The Case of the Function $\boldsymbol{f(x) = \left|x^2 - 5x + 4\right|}$

The first item of part 3 of the question involved recognizing the graphical meanings of multiplying a function with a fraction called stretch transformation; i.e. what graphical change is needed on the graph of the original function as a result of multiplying the function with 1/3. The second requirement of part 3 involved recognizing the appropriated algebraic change in this case.

Answering the first requirement of part 3, seven students recognized the graphical meanings of the algebraic action, drawing the transformed function accurately, as in figure 6 (a). Five students performed the required transformation on one part of the function. Figure 6 (b) shows two examples of students' graphs in this case. Seven students performed the compression transformation instead of the stretch transformation. Figure 6 (c) shows one example of students' graphs in this case.

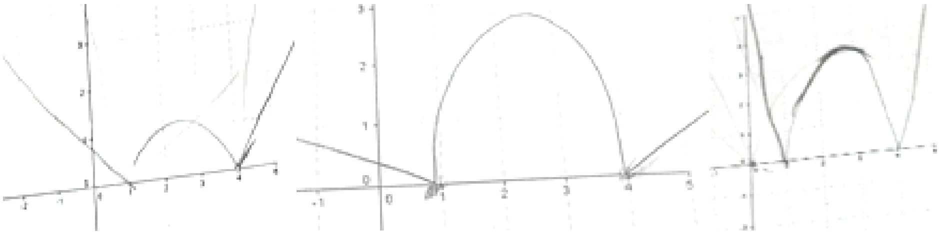


*Figure 6*. Students' recognition of the graphical meaning of the algebraic operations on the original function $f(x) = |x^2 - 5x + 4|$

Answering the second item of part 3, 15 students recognized the algebraic meanings of the algebraic action involved in the stretch transformation. One student performed $k(x) = -1/3|x^2 - 5x + 4|$ the stretch transformation on the reflected function instead of the original one.

**Recognizing the Algebraic, Graphical and Verbal Meanings of the Three Types of Transformation Given Together in the Graphical Representation: The Case of the Function $f(x) = 1/(1 - x^2)$**

Part (a) of the second question examined students' recognition of the algebraic meanings of the three types of transformations given in the graphical representation.

Recognizing the algebraic meanings of the horizontal and vertical translations given in the graphical representation, eleven students recognized the algebraic meaning of the horizontal translation. Five students did not recognize the algebraic meaning of the translation, adding 2 to $x^2$ instead of x (writing $x^2+2$ instead of $(x+2)^2$). One student did not perform the translation transformation, while the last two did not answer the question.

One student recognized the algebraic meaning of the vertical translation. Sixteen students did not recognize the algebraic meaning of the vertical translation. Two of the sixteen students wrote 2 in the numerator of the algebraic expression, instead of adding 2 to it. The rest of the students (14 students) did not recognize the vertical transformation.

Regarding the reflection transformation given in the graphical representation, sixteen students recognized the algebraic meaning of the reflection transformation, while one student did not recognize the algebraic meaning of the reflection transformation, not multiplying the function with (-1). The last two students did not answer the question.

Regarding the stretch transformation given in the graphical representation, fourteen students recognized the algebraic meaning of the stretch translation. Two students did not recognize the algebraic meaning of the translation. The last three students did not answer the question.

We summarize the findings regarding students' recognition of the different transformations on new and no-basic functions, when the algebraic representation is involved. Table 1 shows the number of students who recognized correctly the different transformations.

Table 1

*Recognition of transformations by the participants (n=19)*

| | The function $f(x) = \|x^2 - 5x + 4\|$ | The function $f(x) = 1/(1 + x^2)$ |
|---|---|---|
| Vertical translation | 14 | 1 |
| Horizontal translation | 6 | 11 |
| Reflection | 16 | 16 |
| Stretch | 15 | 14 |

Table 1 shows that students had relative difficulty in performing translations, but were successful with the other transformations.

## Discussion and Conclusions

Previous studies' results indicate that school and college students have problems in performing function transformations in general and transformations on new functions in particular (Eisenberg & Dreyfus, 1994; Zazkis, Liljedahl & Gadowsky, 2003). This research came to examine how new technology, in our case Geogebra, impact students' conceiving of non-basic and new functions.

The findings as described in table 1 show that the participating students generally were able to work successfully with transformations on new and non-basic functions, though they encountered difficulties in working with translations. These findings are discussed in more detail below.

The students could work relatively successfully with transformation on new and non-basic functions due to the potentialities of Geogebra, where it influences positively students' learning of mathematics (Goldin & Shteingold, 2001; Zbiek, Heid, Blume & Dick, 2007; Reisa, 2010). It does that by enabling student to illustrate mathematical objects (in our case functions and function transformations), and connect among the various mathematical representations. Moreover, it extends the possible objects that the learner can work with (in our case, the basic functions could be extended to non-basic ones), and make generalizations through investigation and experimenting.

Furthermore, different researchers and mathematical organizations (NCTM, 2000; Noss, Healy & Hoyles, 1997) pointed at the importance of connecting between the symbolic and visual representations of mathematical concepts, which contributes to students' understanding of these concepts more deeply. Geogebra enabled the described connection that supported the students in their working with, and thus understanding transformations.

Recognizing the algebraic meanings of verbal expressions associated with translations, more students recognized the algebraic meaning of the vertical translation than those recognizing the algebraic meaning of the horizontal translation on the function $f(x) = |x^2 - 5x + 4|$(14 students versus 6 students).

Researchers gave different explanations to justify this phenomenon. For example, Eisenberg and Dreyfus (1994) suggested that the horizontal transformation involves more visual processing than the vertical transformation, while Baker, Hemenway and Trigueros (2000) suggested that the horizontal translation is more complicated cognitively than the vertical transformation because the vertical transformations are actions performed directly on the basic functions, while horizontal transformations involve two mental actions: the first action is performed on the independent variable of the function, while a second action is needed on the object resulting from the first action to get the transformed function, or to absorb the mathematical idea behind the horizontal transformation. In the above specific function (absolute value of the quadratic function), students' difficulties in performing horizontal translation on the function could also be attributed to the variable x appearing twice in the algebraic expression of the function, a situation that the students have not been accustomed to before.

In contrast to students' difficulty with the horizontal translation in the case of the function $f(x) = |x^2 - 5x + 4|$, the students had difficulty with the vertical translation on the function $f(x) = 1/(1 + x^2)$. Previous studies did not report this difficulty. Nevertheless, this phenomenon could be attributed to the form of the algebraic expression of the function, where the function is a rational function whose denominator is composed not only of a variable but a number too. This form is different from all the functions that the students worked with, where the original function was composed of just the variable. Furthermore, Students' difficulties reported above regarding performing translations could be overcome by giving the students varied types of functions, for example functions which include the x variable more than once, or rational functions that has numbers in their denominator.

It can be concluded that the current research findings show that the participants were generally able, after learning the transformation topic with the help of technology (in our case GeoGebra), to solve successfully mathematical problems, by means of function transformations, involved with new and non-basic functions. Furthermore, the participants encountered difficulties in working with translations. These difficulties were due to the special algebraic form of the functions (the variable x appearing twice or appearing with a number in the algebraic rule of the original function). It is recommended that middle school students would be exposed to transformations on these functions during learning this mathematical topic. Future researches could examine the impact of activities that include such functions and that are GeoGebra based on students' conceptions and behavior when performing translations is involved.

## References

Baker, B., Hemenway, C., & Trigueros, M. (2000). On transformations of basic functions. In: H. Chick, K. Stacey, & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference on the Future of the Teaching and Learning of Algebra* (Vol. 1, pp. 41–47). University of Melbourne.

Consciência, M. & Oliveira, H. (2011). Function concept and functions' transformations: The role of the graphic calculator. CERME7, February 7-13, 2011, Rzeszów, Poland.

Eisenberg, T., & Dreyfus, T. (1994). On understanding how students learn to visualize function transformations. In: E. Dubinsky, A. Schoenfeld, & J. Kaput (Eds.), *Research in collegiate mathematics education* (Vol. 1, pp. 45–68). Providence, RI: American Mathematical Society.

Goldin, G. A., & Shteingold, N. (2001). Systems of representations and development of mathematical concepts. In J. Cuoco (Ed), *The roles of representation in school mathematics* (pp. 1-22). Reston, VA: NCTM.

Kimani, (2008). *Calculus students' understandings of the concepts of function transformation, function composition, function inverse and the relationships among the three concepts*. Unpublished doctoral dissertation, Syracuse University, Syracuse, NY.

Lage, A. E. & Gaisman, M. T. (2006). An analysis of students' ideas about transformations of functions. In A. S., Cortina, J.L., Saíz, and Méndez, A.(Eds), *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 68-70). Mérida, México: Universidad Pedagógica Nacional.

McClaran, R. R. (2013). *Investigating the impact of interactive applets on students' understanding of parameter changes to parent functions: an explanatory mixed methods study*. Unpublished PhD dissertation. Retrieved from http://uknowledge.uky.edu/cgi/viewcontent.cgi?article=1001&context=stem_etds

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Noss R., Healy L., Hoyles C. (1997). The construction of mathematical meanings: Connecting the visual with the symbolic. *Educational Studies in Mathematics, 33*, 203-233. doi: 10.1023/A:1002943821419

Reisa, A. Z. (2010). Computer supported mathematics with GeoGebra. *Procedia - Social and Behavioral Sciences, 9*, 1449- 1455. doi:10.1016/j.sbspro.2010.12.348

Sever, G. & Yerushalmy, M. (2007).To sense and to visualize functions: The case of graphs' stretching. In P. P. Demetra & P. George (Eds.), *The Fifth Conference of the European Society for Research in Mathematics Education* (CERME5) (pp. 1509-1518). Larnaca: Department of Education, University of Cyprus.

Zazkis, R., Liljedahl, P. & Gadowsky, K. (2003). Conceptions of Function Translation: Obstacles, Intuitions, and Rerouting. *Journal of Mathematical Behaviour, 22*, 437–450. doi: 10.1016/j.jmathb.2003.09.003

Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: a perspective of constructs. In Frank K. Lester (Ed.). *Second Handbook of Research on Mathematics Teaching and Learning: A project of the National Council of Teachers of Mathematics* (Vol. II, pp. 1169-1207). Charlotte: Information Age Publishing.

Zhang, Y., & Wildemuth, B. M. (2009). Qualitative analysis of content. In B. Wildemuth (Ed.), *Applications of Social Research Methods to Questions in Information and Library Science* (pp.308-319). Westport, CT: Libraries Unlimited.

**Wajeeh Daher** is an Associate Professor of Mathematics Education in in An-Najah National University, Palestine, and Senior Lecturer in Al-Qasemi Academic College of Education, Israel.

**Ahlam A. Anabousy** is an Teaching Assistant of Mathematics in Al-Qasemi Academic College of Education, and in Tel-Aviv University, Israel.

**Contact Address:** Direct correspondence concerning this article, should be addressed to the author. **Email**: wajeehdaher@gmail.com